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# Co-Optimal PMU and Communication System Placement Using Hybrid Wireless Sensors

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## Abstract

In smart distribution networks, the problem of measuring and estimating the state of the system is vital. Communication infrastructure, as an inseparable part of the wide area measurement system (WAMS), has to be optimally designed and placed to collect information from phasor measurement units (PMUs) and deliver them to control centers. In this paper, a novel hybrid wireless sensor network (WSN) is proposed for connecting of PMUs throughout the system to enable a convenient and low cost communication media. Having found the optimal placement of PMUs in power system to reach a full observability, the problem of observability in communication system will be handled to provide proper transmission of measured data to the control centers. In the proposed co-optimal PMU-sensor placement problem, the main objective is to minimize the total cost of PMU placement in power system and related communication system considering full observability and reliability constraints. For increasing the reliability of communication system, a hybrid WSN including plug-in powered sensor nodes (PPSNs) and energy harvesting sensor nodes (EHSNs) is utilized. A binary-coded genetic algorithm (BCGA) is used for solving the proposed mixed-objective optimization problem subject to different technical operating constraints. The proposed method is examined on a sample 11-bus test system and IEEE 37-bus test feeder systems. The results show the applicability and effectiveness of the proposed algorithm compared to the conventional methods in this subject area.

**Keywords:** Phasor measurement unit (PMU), communication system, observability, plug-in powered sensor node (PPSN), energy harvesting sensor node (EHSN).

## 1. Introduction

The emerging power distribution systems with a proliferation of different means of dispersed generation (both conventional and renewable) and flexible demand assets is expected to undergo a restructuring process, just as what has been happening in the transmission system [1]. In order to assure proper operation, control and planning of such systems, measurement of the system states is unavoidable [2]-[3]. In fact, in order to well estimate the state of the system, observability is a must [4]. Phasor measurement units (PMUs) have been used extensively for measuring voltage and current in power systems and are deemed as key players in wide area measurement systems (WAMS) [5]. To make a fully observable system while being cost-effective, the number of required measuring devices in the network must be optimized [6].

In recent years, many methods have been proposed to optimize the PMU placement with regard to the observability of the system. Most of these methods have tried to minimize the number of measuring devices to reduce the cost of the WAMS [7]-[8]. The proposed methods for solving the optimal PMU placement (OPP) can be divided into two groups: mathematical methods and heuristic methods. All of these methods aim to minimize the number of PMUs that guarantees the full observability considering several scenarios such as contingency [9]-[11], redundancy [12]-[13], and data availability [14]. However, most of these methods did not consider the communication cost associated with PMU placement problem [15]. In fact, individual optimal placement of

PMUs and communication infrastructures cannot provide a global optimal solution to WAMS [16]. In [17], optimal PMU-communication link placement has been proposed and demonstrated that co-optimal placement could end in a system-wide optimality. In [18] the optimal placement of phasor data concentrator (PDC) has been added to the objective function and a combined approach based on binary imperialistic competition algorithm (BICA) and Dijkstra algorithms has been used to solve the optimization problem. Likewise, a PMU-communication problem has been solved in [19] using the variable neighborhood search (VNS) algorithm.

Some other researches focus on bandwidth cost of communication system such as [20] where authors included the effect of zero-injection buses (ZIBs) and bandwidth cost into the optimal PMU-communication link placement. They showed that because of the bandwidth cost, the shortest communication link may not necessarily lead to the total minimum cost.

In [21], the authors proposed a microwave communication technology instead of the conventional fiber-based links with the aim of minimizing the propagation delay in optimal placement of PMUs, PDC and communication links. A method for optimal PMU placement has been introduced in [22] where all the possible solutions of PMU placement have been determined initially using the binary-coded genetic algorithm (BCGA). For the search improvement of BCGA a branch-and-bound algorithm is used. Two-phase branch-and-bound algorithm is proposed in [23] to guarantee the global solution of optimal PMU placement. In [24], a CPU time comparison between branch-and-bound algorithm and BCGA is presented and multiple solutions found by these algorithms are discussed.

In [25], a binary semi-definite programming model with binary decision variable is proposed for consideration the effect of conventional measurement and zero injections on OPP with respect to the numerical observability. Authors in [26] implement the optimal  $\mu$ -PMU placement for smart distribution networks using a binary integer linear programming (BILP) model. They consider the effect of fully ZIBs and partially ZIBs in distribution network observability. Also, an optimal meter placement in distribution network by minimization of state estimation error is proposed in [27].

In most of the reviewed literature, researches have assumed that the communication links are based on conventional optical fiber cables. The fiber-link communication needs a major infrastructure, thus a very high investment cost would be inevitable. As an alternative, wireless sensors can be effectively used to yield a low-cost communication system with a simple infrastructure.

However, for proper implementation of a wireless sensor network (WSN), the reliability of communication is very important. Today, wireless sensors are extensively used in different applications such as medical care [28], military [29], environment monitoring [30], various industries automation [31], civil [32], and home security [33]. Most conventional wireless sensors are driven by batteries. These battery-powered sensor nodes (BPSNs) incorporate a physical power switch, which is mounted on the circuit board. Given the fact that switching or replacing BPSNs is very difficult or even impossible in some cases; their energy consumption is a very important issue. A lot of research has been done to minimize the energy consumption of these sensors or to maximize battery lifetime [34]-[35]. Energy harvesting sensor nodes (EHSN) can be used to solve the lifetime problem of battery-based sensors. EHSN can harvest energy from the environmental sources such as vibration, solar energy, and wind. The harvested energy can be converted into usable electrical energy for wireless sensors [36]. It should be noted that due to the limitations on harvested energy, EHSNs must also be optimized.

Generally, the reliability of wireless sensors can be considered in two parts: sensor node reliability and wireless link reliability. The reliability of sensor nodes has been detailed in [37]-[38]. The link reliability of BPSNs is time-dependence meaning that their reliability reduces gradually as time passes. However, in plug-in powered sensor nodes (PPSNs) and EHSNs, if energy sources are sufficient, their reliability will depend on the distance to the next sensor [39]-[40].

In this paper, the optimal PMU placement together with the needed communication links using a combination of PPSN and EHSN is proposed. The objective function includes the total cost minimization and reliability maximization. Cost of this system relates to PMU cost and communication system cost. To achieve better results, the power network and communication system are optimized simultaneously. A BCGA is used for solving the proposed optimization problem considering three major constraints: communication constraint and observability constraints both from the power system (PMUs) and communication network (WSNs) perspectives. Also, the third constraint is the reliability of communication system.

As a whole, the main contributions of this study can be summarized as follows:

- An efficient and low-cost communication infrastructure is proposed based on PPSNs and EHSNs to enable secure communications among PMUs and the control center
- A method for communication observability is introduced for applications in hybrid WSNs
- A reliability analysis is presented to evaluate the reliability budget impact on optimal PMU-sensor placement.

The rest of this paper is organized as follows. A definition of PMUs and WAMS is presented in section II. The problem formulation and the proposed optimization method are considered in section III. The numerical study and simulation results are carried out in section IV. The conclusion is in section V.

## 2. Definition and application of WAMS and PMUs

The measurements performed in the supervisory control and data acquisition (SCADA) system are not usually synchronous leading to a timing difference among measured data. On the other hand, sampling rate is not high in measuring devices, thus, the received information by the SCADA system, in the most optimistic way, displays the system's steady state leaving no room to monitor the system's dynamic condition. Along with the SCADA system, WAMS has been developed to compensate for the shortcomings of the SCADA system.

PMUs are the main component of the WAMS that can measure the voltage and current phasors with high precision (less than 0.1% error) and high speed (up to 60 samples per second). PMUs use the global positioning system (GPS) to synchronize measured data. If enough PMUs are installed in different buses, the system will be observable and system operator is able to estimate the state of the system [41].

Since PMUs could measure the voltage and current phasors simultaneously, it is not necessary to install PMUs in the entire system. Therefore, one of the important issues is finding the optimal number and location of the PMUs to have a fully observable system while being economic and viable in practice.

Figure 1 illustrates how to calculate the voltage and current of the nodes and branches in a 4-bus system using available data from two PMUs. As can be seen, using the measured data by PMUs, the states in adjacent buses can be easily calculated as follows:

$$V_2 \angle \phi_{v2} = V_1 \angle \phi_{v1} - Z_1 * I_1 \angle \phi_{I1} \quad (1)$$

$$V_3 \angle \phi_{v3} = V_4 \angle \phi_{v4} + Z_3 * I_3 \angle \phi_{I3} \quad (2)$$

$$I_2 = \frac{V_2 \angle \phi_{v2} - V_3 \angle \phi_{v3}}{Z_2} \quad (3)$$

$$I_{L2} \angle \phi_{IL2} = I_1 \angle \phi_{I1} - I_2 \angle \phi_{I2} \quad (4)$$

$$I_{L3} \angle \phi_{IL3} = I_2 \angle \phi_{I2} - I_3 \angle \phi_{I3} \quad (5)$$

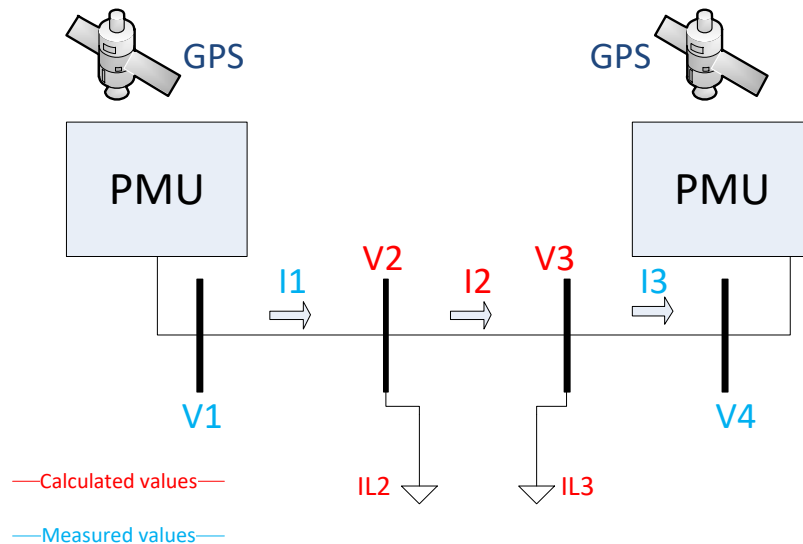


Fig. 1 An example for calculating the system nodal voltages and branch currents using measured data from PMUs

It is worth mentioning that at distribution level,  $\mu$ PMUs can also be used to enable wide-area measurements. Such devices offer a more economic solution at a expense of lower response to the events and less accurate phasor angel computation compared to PMUs [42].

### 3. Problem formulation

In this paper, optimal PMU placement together with the needed sensor-based wireless communication system is studied. The aim is to minimize the number of measurement devices and related communication infrastructures while meeting system's constraints. To achieve this goal, it is necessary to model the components of the power and communication systems appropriately. In this section, the optimal PMU placement problem is initially defined and formulated mathematically. Then, the communication system requirements are discussed. In the next step, the observability and reliability aspects of the WSN are presented. Finally, the structure of the proposed solution method based on binary-coded genetic algorithm is described.

#### 3.1. OPTIMAL PMU PLACEMENT

As mentioned earlier, due to the high price of PMUs, it is not economically viable to equip all buses with such devices. The general objective for optimizing the placement of PMUs in power grid is using the least number of measuring devices corresponding to the minimum cost, and establishing the observability of the network with the mentioned measurements.

A power grid is fully observable if all of its buses (i.e. nodes) can be observed. Likewise, by definition, a node in power system is observable if that node or any of its neighboring nodes (which are connected to the target node) is equipped with a PMU. In other words, the presence of a PMU at a bus is adequate for observability of all adjacent buses.

If the number of network buses is equal to  $N$ , then the matrix of the nodes will be  $N \times N$ , which is formed as follow:

$$A_{i,j} = \begin{cases} 1 & (i=j) \text{ or } (i \text{ is connected to } j) \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Moreover, the existence of a PMU in a bus can be represented mathematically as:

$$x_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Accordingly, the optimal PMU placement problem can be derived as follows:

$$\text{Min } CX \quad (8)$$

$$\text{s.t. } AX \geq u \quad (9)$$

Where,  $C_i$  is the cost of purchasing and installing PMU at bus  $i$ .

$$C = [C_1, C_2, \dots, C_N] \quad (10)$$

The  $X$  vector contains all the  $x_i$  variables and  $u$  is the unit matrix.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_N \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ 1 \end{bmatrix} \quad (11)$$

Based on (9), a system will be fully observable if each node/bus can be observable by at least one PMU.

### 3.2. COMMUNICATION SYSTEM

As mentioned before, the WSN is used as means of communication in this research. Such network includes a number of sensor nodes, each has special capabilities such as environmental conditions measuring, storing and processing information as well as wireless communications with adjacent nodes [38]. A given sensor node consists of five elements: sensor unit, processing unit, receiving and sending unit (transmitter / receiver), energy management unit or power supply unit, and eventually accessory equipment. Moreover, each sensor node in the network has to send the collected information by the sensor itself and the received information from the adjacent nodes to the sink node (i.e., control center) in the role of a router.

Most conventional wireless sensors are battery driven. Given the fact that switching or replacing these sensors is very difficult or almost impossible; their energy consumption is a very important issue. EHSN can be used to solve the lifetime problem of battery-based sensors. EHSN can harvest energy from environmental sources such as vibration, solar energy and wind. The harvested energy can be converted into usable electrical energy for wireless sensor consumption. Due to new EHSNs technology, some types of these sensors can be driven by both batteries and energy-harvesters, which greatly improve the communication reliability. So the lifetime of this sensors are modified.

In this paper, the connection of PMUs is performed using combination of PPSNs and EHSNs. To achieve this goal with the lowest cost and highest reliability, the network is divided into two categories: the nodes that measured by PMUs, and the router nodes. In the first group, PPSNs are used as the sensor can receive the required energy from the PMUs. The second group benefits from the EHSNs to assure longer lifetimes. By doing so, the reliability of the system can be improved.

As illustrated in Figure 2, the connection among PMUs should be made through EHSNs. Thus, the minimum number of sensors and a maximum reliable path for transferring the measured data to the control center must be found. In this research, the location of control center is determined and fix. Here, a number of assumptions are made: first, it is assumed that wireless sensors can transfer the PMUs data successfully. Second, the bandwidth of wireless sensors is supposed to be sufficient for transferring measured data by PMUs.

With the mentioned assumptions, the structure of communication system can be well constructed. However, because of using the wireless sensors as communication system, the observability analysis from the viewpoint of communication system is necessary. This issue will be considered in the next section.

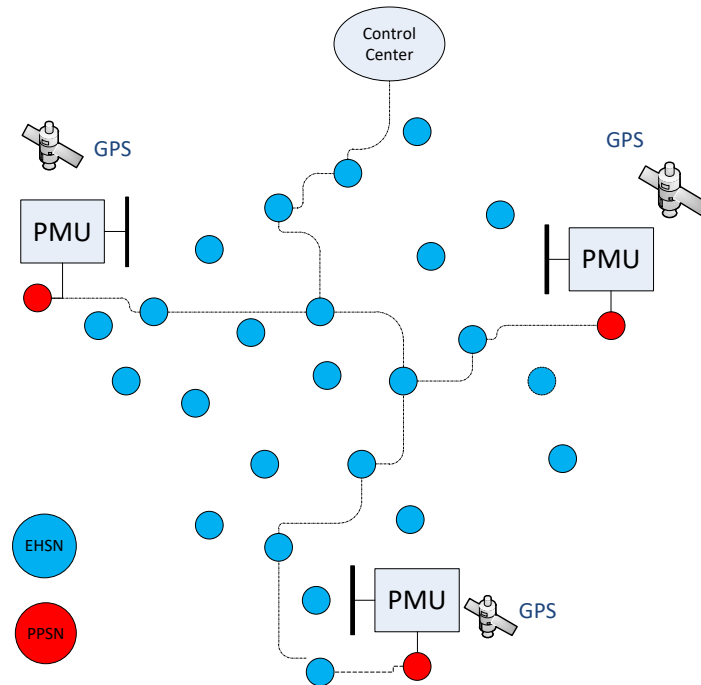


Fig. 2 Communication system

### 3.3. COMMUNICATION SYSTEM OBSERVABILITY

After optimal placement of PMUs, data has to be transferred successfully to the control center which demands a fully-observable communication system. In the conventional systems based on optic fiber cables, it is not necessary to analyze the observability as the cables interconnect the entire nodes. But in wireless communication systems, similar to the OPP, the observability analysis has to be done to be sure that wireless sensors can communicate to each other and to the control center. Thus, the definition of observability in communication system is presented as follow.

**Definition:** A node in communication system is observable if there is a connection between that node and the reference node.

The reference node is a node that the data should be ultimately delivered to. The reference node in this research is the control center. In other words, the wireless node is observable if it is not islanded.

In order to make the communication network observable by wireless sensors, a mesh grid of power network is initially extracted. In this way, the geographical area (where the distribution network is located) is divided into a set of equal areas/meshes as shown in Figure 3. Wireless sensors can be placed in any area. However, due to the communication range of wireless sensors, the mesh sizes must be selected in a way so that the presence of a sensor in an area could cover the observability of the adjacent areas. According to Figure 3, the relationship between the transmitting range ( $tr$ ) of the wireless sensors and the length of each area is obtained as (12).

$$r = \frac{\sqrt{2}}{2}d \rightarrow tr = 3r = \frac{3\sqrt{2}}{2}d \rightarrow d = \frac{2}{3\sqrt{2}}tr \quad (12)$$

So if the transmitting range of a wireless sensor is specific, the mesh size (i.e., length of each area ( $d$ )) is obtained.

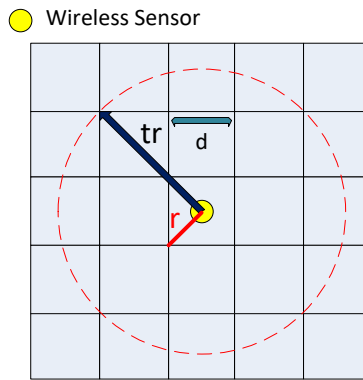


Fig. 3 Transmitting range of wireless sensor and length of mesh grid relation

If the length and width of a geographical area in which the communication system has to be set up, is equal to  $W_1$  and  $W_2$ , respectively, the number of transverse ( $n_1$ ) and longitudinal ( $n_2$ ) areas in the mesh grid is as:

$$\begin{aligned} n_1 &= \left\lceil \frac{W_1}{d} \right\rceil + 1 \\ n_2 &= \left\lceil \frac{W_2}{d} \right\rceil + 1 \end{aligned} \quad (13)$$

Accordingly, total number of meshes is equal to:

$$N_T = n_1 \times n_2 \quad (14)$$

Figure 4 illustrates the difference between observability from power system and communication network perspectives. As can be seen, the examined 7-bus power system could be observable through placement of two PMUs in buses 1 and 4. However, if the distance between buses 1 and 4 be greater than the transmission range ( $tr$ ) of PPSNs, the communication system is unable to

transmit data to the center, thus will not be observable. To make the communication system observable, a step-by-step procedure is presented through an illustrative example. Let's assume that  $tr = 100m$ . The mesh size ( $d$ ) can be calculated according to (12).

$$d = tr \frac{2}{3\sqrt{2}} = 47.6m \quad (15)$$

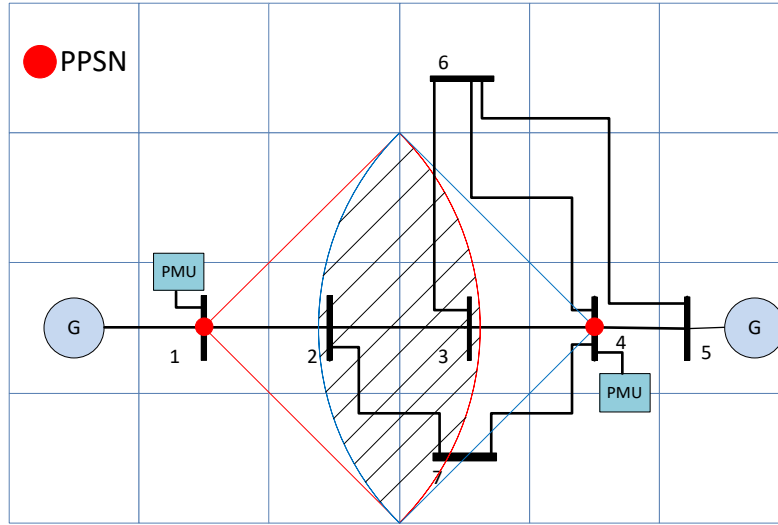


Fig. 4 A 7 bus test system for communication system observability analysis

Given the length and width of the geographical area over which the communication system has to be set up,  $n_1$  and  $n_2$  can be calculated based on (13) and the mesh grid is accordingly created (Figure 4). As can be seen, a router sensor is needed in the intersected area to make the communication network observable. As can be seen in Figure 4, buses 2 and 3 are located in the common transmission range of the two sensors. Thus one of these nodes can be chosen as a router node. Another sitting can also be adopted to make both systems observable. By moving one of the PMUs (e.g., PMU from bus 1 to bus 2), not only the power system would remain observable, but also observability of the communication system can be achieved without using a router since the distance between the two sensors becomes less than  $tr$  and they could communicate to each other directly. In larger networks, however, such sittings cannot be realized easily and an optimization problem has to be solved to find optimal placement of PMUs and wireless sensors with respect to observability constraints.

To do so, BCGA is implemented in this study for optimal placement of wireless sensors following the optimal placement of PMUs based on power system observability constraint. The chromosome (CR) is coded in such a way to reflect the existence and position of the primary sensor units. It is supposed that in each place that a PMU exists, a PPSN can be placed. So the number of elements in CR is equal to the total number of meshes ( $N_T$ ) while each could be assigned a binary value (1: having PPSN and 0, otherwise):

$$CR = [SN_1 \ SN_2 \ SN_3 \ \dots \ SN_n]_{1 \times N_T} \quad (16)$$

$$SN_i = \begin{cases} 1 & \text{with PPSN} \\ 0 & \text{without PPSN} \end{cases}$$

SN is the existence of sensor node in each area. As an example, the following vector shows an initially coded chromosome of BCGA for the system described in Figure 4:

$$CR = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]_{1 \times 24} \quad (17)$$

Having formed the initial population, the distance matrix has to be determined as follows:



$$L = \begin{bmatrix} L_{11} & L_{12} & \dots & L_{1n} \\ L_{21} & L_{22} & \dots & L_{2n} \\ \dots & \dots & \dots & \dots \\ L_{n1} & L_{n2} & \dots & L_{nn} \end{bmatrix} \quad (18)$$

Where  $L_{ij}$  is the distance of area  $i$  from area  $j$  in the mesh grid. Two areas  $i$  and  $j$  are in direct connection with if the following conditions are satisfied:

- 1- Both areas have wireless sensors ( $tr_i \neq 0$  and  $tr_j \neq 0$ )
- 2- The wireless node of one region is located in the transmitting range of the other area ( $tr_i \geq L_{i,j}$  or  $tr_j \geq L_{i,j}$ )

In order to determine the connections among areas, a matrix is formed that shows the connection of different regions with each other. This matrix is called the incident matrix (IM) and can be defined as:

$$IM_{i,j} = CR_i \times CR_j \times (tr_i \geq L_{i,j}) \times (tr_j \geq L_{j,i}) \quad (19)$$

The matrix size is  $N_T \times N_T$ . If the number of network buses is  $N$ , the communication observability (CO) matrix can be used for observability analysis of communication system [43]. This matrix can be defined as:

$$CO = sign(IM^{N-1}) \quad (20)$$

After determination of control center, if all nodes are in contact with the control center, the communication system is observable. Matrix CO shows the connectivity of the nodes to the control center. If control center is in area  $S$  and the set of all communication nodes be  $Z$ , for complete observability of communication system the relation (21) should be true.

$$CO_{s,j} = 1 \quad \forall j \in Z \quad (21)$$

This is the communication constraint of the optimization problem. To achieve the lowest cost, the least number of router sensors (which are EHSNs in this study) must be used. Thus, an optimization process to determine the minimum sensor number should be formulated.

### 3.4. COMMUNICATION SYSTEM RELIABILITY

Reliability of the WSNs consists of two parts: sensor node reliability and wireless link reliability. In this paper for the sake of simplicity, only the reliability of the wireless link is considered. Each wireless sensor  $i$  has a transmitting range ( $tr_i$ ). The transmitting range is the distance that the sensor can see other sensor nodes. The wireless link reliability is defined as the probability of having a reliable link between the EHSNs and PPSNs. This value can be defined as a function of distance between the receiver and transmitter [36]:

$$P_{link} = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{10 \log(d / tr)}{\sqrt{2} \log(10\psi)} \right) \right] \quad (22)$$

$$\psi = \frac{\sigma}{\eta} \quad \text{if } \frac{d}{tr} \leq 1$$

Where,  $\sigma$  is the standard deviation of shadowing,  $\eta$  denotes the pass loss exponent,  $\psi$  is the strong or weak relation of shadowing effect,  $\text{erf}$  is the error function and  $d$  is the distance between two sensors.

As can be observed, with increasing the distance between two sensors, the reliability of transferring data is decreased. If the distance ( $d$ ) between two sensors increases beyond the  $tr$ , reliability will be zero. The relation between the distance of two sensors and link reliability is illustrated in Figure (5). In the figure it is clear that if the relation of  $d/tr$  is greater than about 0.6, the

reliability decreases exponentially. Therefore, in order to increase the reliability of the system, the distance between sensors must not be allowed to reach the upper band limit.

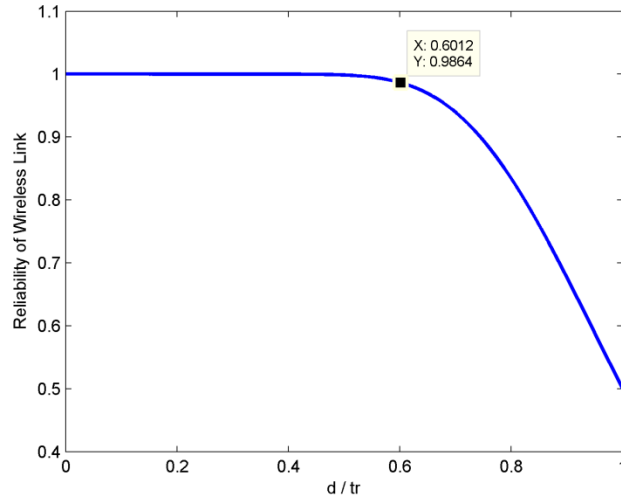


Fig. 5 Wireless link reliability in EHSN [32]

Therefore, in this paper, the wireless link reliability of PPSNs and EHSNs are same and can be evaluated by (22).

### 3.5. PROPOSED OBJECTIVE FUNCTION

In the previous sections, the examined model of the power system and communication system was presented. In the WAMS, the cost of communication infrastructure is as important as PMU placement. So, in the proposed method, the communication system and PMU placement is simultaneously optimized. Cost function includes the minimization of communication system cost and PMU placement cost with respect to the communication, reliability and observability constraints.

In this paper, total cost can be defined as below:

$$Total\ Cost = Cost_{PMU} + Cost_{WSN} \quad (23)$$

Where  $Cost_{PMU}$  is the total cost of optimal PMU placement in the network and  $Cost_{WSN}$  is the cost of sensors that is necessary for successful data transmission. However, increasing the reliability of the system will increase the costs. Therefore, as explained in the previous section, the reliability of the system will be presented as a constraint in the optimization problem. Depending on how important the reliability is, the constraint could be tightened. So the objective function can be written as below:

$$Min \sum_{i=1}^N c_i X_i + P_p NP + P_e NE \quad (24)$$

$$St. \quad AX \geq 1 \quad (25)$$

$$CO_{n-1,j} = 1 \quad j = 1, 2, \dots, m \quad (26)$$

$$Re_j \geq T \quad j = 1, 2, \dots, m \quad (27)$$

where,  $C_i$  represents the cost of PMU in bus  $i$ ,  $X_i$  is the status of installing or not installing the PMU in that bus,  $N$  is the number of buses,  $P_p$  is the cost of PPSN while  $NP$  denotes the number of PPSNs. In a same way,  $P_e$  is the cost of EHSN and  $NE$  is the number of EHSNs.  $Re_j$  is the reliability of wireless link in sensor  $j$ .  $m$  is the total number of wireless sensors.

Equation (25) represents the observability of power system that is described in section 3.1. Equation (26) describes the communication system observability that is defined in section 3.3. Reliability constraint is represented in (27). Given the

importance level of reliability, the right hand side of (27) could be adjusted in a range from 0 to 1.  $T$  is a coefficient that shows the impact of reliability. For example, if the reliability should be very high, the  $T$  coefficient is set to one. Flowchart of the proposed method is illustrated in Figure 6. As a whole three main steps should be followed to implement the approach.

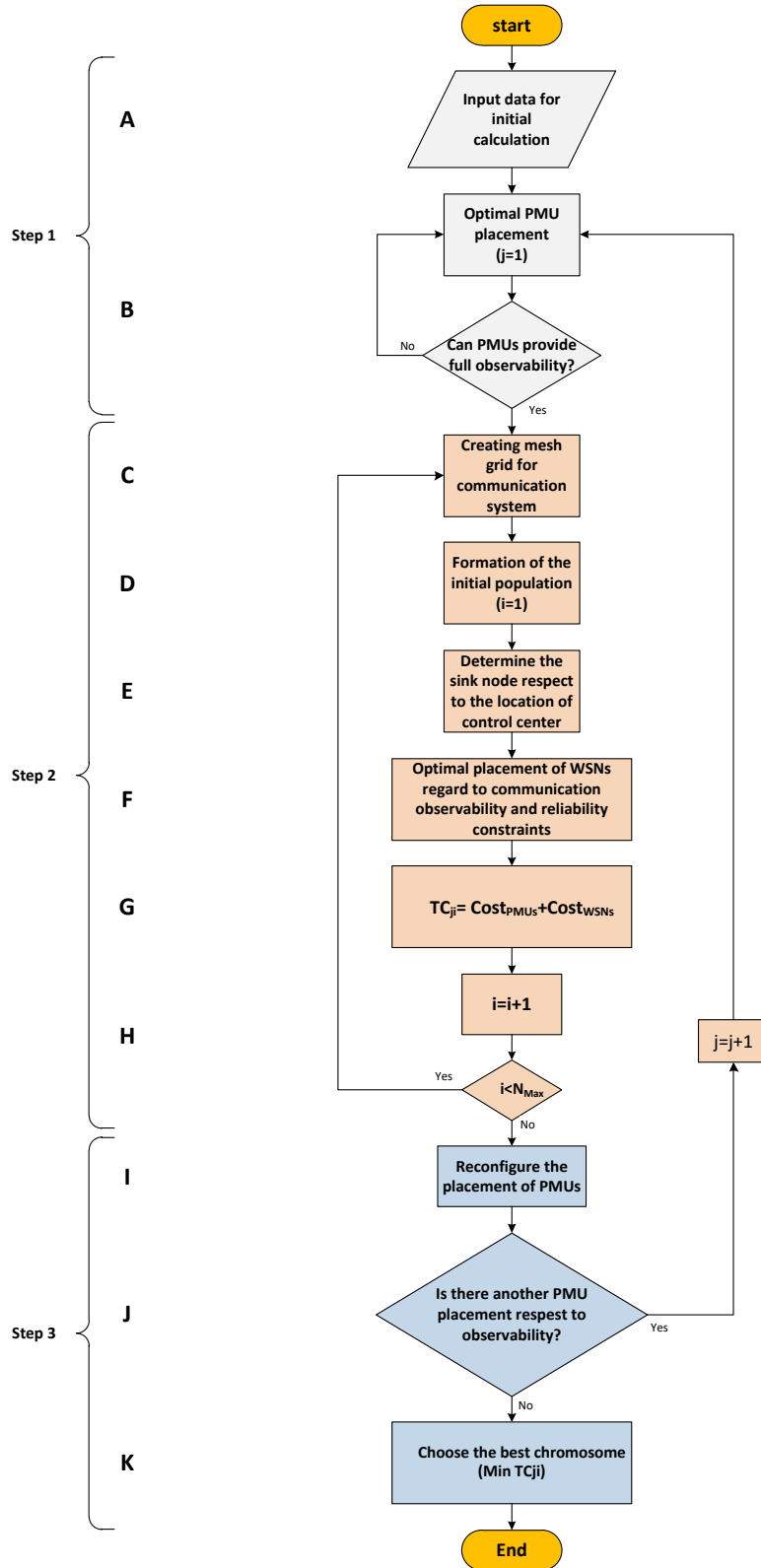


Fig. 6 Flowchart of proposed method

At the first step, OPP with regard to the power system observability is calculated. At the second step, optimal placement of wireless sensors (PPSNs and EHSNs) with respect to the communication system observability and reliability constraints is considered. In the last step, the best cost-effective solution is determined. A detailed review of the aforementioned steps is given below.

**Step 1:** the input data (i.e., number of buses, distances and connections) is checked in block A. OPP with respect to observability constraint is implemented in block B using (8)-(9). It is noteworthy that more than one solution might be found at this stage.

**Step 2:** in block C, the mesh grid for proper placement of wireless sensor is formed according to (12)-(15). Based on the optimal PMU placement results, the initial population is formed in block D considering (16). To transfer the data to the control center, a wireless sensor as the sink node should be selected which is done in block E. It is assume that the location of control center is determined and fix. In block F, the communication cost (Eq. (24)) is minimized with respect to communication observability (Eq. (26)) and reliability (Eq. (27)) constraints. The total cost of each possible solution will be determined in block G and will be stored in repository of optimal solutions. Block H represents the stop criterion (which is set to a maximum number of iterations) for the BCGA.

**Step 3:** for achieving the best solution all possible solutions stored in the repository should be evaluated. As noted before, OPP usually can be done in multiple ways and each plan of action has a related communication cost. In the last step, after optimization of wireless sensor placement in all the possible cases, the best solution that is based on the minimum total cost is reported.

## 4. Case study

In this section, the proposed method is implemented on sample networks. The aim of this study is to examine the advantages and feasibility of using wireless sensors as communication system in WAMS. For proper testing of the proposed method, the sample 11-node and IEEE 37-node test systems are utilized. In each case, BCGA parameters are set (calibration was done by trial and error) as reported in Table 1 where,  $P_C$  is the probability of crossover and  $P_M$  is the probability of mutation. Three formats of crossover could be selected for the next generation; single point crossover, double point crossover and uniform crossover. The exploration and exploitation ability of these methods are different for new generation. In this paper, all the aforementioned formats have been used with a probability of selection. The probability of selection setting could be varied. The mutation function is defined by uniform mutation. In order to define the elites of every operator, the best individuals of every generation are selected to be used in the next round.

The population size is also a tradeoff between the exploration capability and computational burden [44]. In this paper, the population size is selected by running the program and tracking the ability to obtain the global solution and satisfying the constraints.

TABLE I. BCGA PARAMETERS FOR OPTIMIZATION

<i>network</i>	<i>Population size</i>	<i>P<sub>C</sub></i>	<i>P<sub>M</sub></i>
Sample 11 node	100	0.8	0.3
IEEE 37 node	150	0.75	0.35

### 4.1. SAMPLE 11-NODE TEST FEEDER

The schematic of this test system is illustrated in Figure 7. The network line segment data is listed in Table 2. A modified BCGA is used to determine the multiple solutions of OPP which is configured as a mixed integer linear programming (MILP) model [45]. To validate the uniqueness of the multiple optimal solutions found by BCGA, a number of techniques can be used. However, in this work, the same procedure adopted in [46] and [47] is used. Assume that  $X_{s,1}=[X_{s,1,1} \ X_{s,1,2} \ \dots X_{s,1,n}]$  is an optimal solution and  $m$  is the optimal number of PMU required. In order to avoid reporting duplicate solutions, the inequality  $X_{s,1}^T \cdot X_{s,2} \leq m-1$  should be true if  $X_{s,2}$  is a different solution from  $X_{s,1}$ . If  $X_{s,2}$  is the same solution as of  $X_{s,1}$ , then  $X_{s,1}^T \cdot X_{s,2} = m$ .

There are two possible solutions for this network considering the observability constraints that is shown in Table 3. Using the simultaneous optimization process (Eqs. (24)-(27)), the best position of PMUs and wireless sensors is obtained with regards to the communication and power systems observability and reliability constraints. According to section 3.2, there is one PPSN in each bus where a PMU is installed. Based on configuration of PMUs tabulated in Table 3, power system is observable in two different ways; however, none of them ensures observability in communication network. Therefore, it is necessary to use some EHSNs as routers

for transferring data to the control center. Node 4 is chosen as the control center in this case. It is assumed that the transmitting range ( $tr$ ) of each PPSN and EHSN are 150 meters. The length and width of the communication network over that the sensors must be placed are about 1315m and 592m, respectively. With these data, the mesh grid for communication observability analysis is constructed using (12)-(14). The calculated values are:

$$\begin{aligned}
 d &= 70.17 \text{ m} \\
 n_1 &= \left\lceil \frac{W_1}{d} \right\rceil + 1 = \frac{592}{47.6} + 1 = 9 \\
 n_2 &= \left\lceil \frac{W_2}{d} \right\rceil + 1 = \frac{1315}{47.6} + 1 = 19 \\
 N_T &= n_1 \times n_2 = 171
 \end{aligned} \tag{28}$$

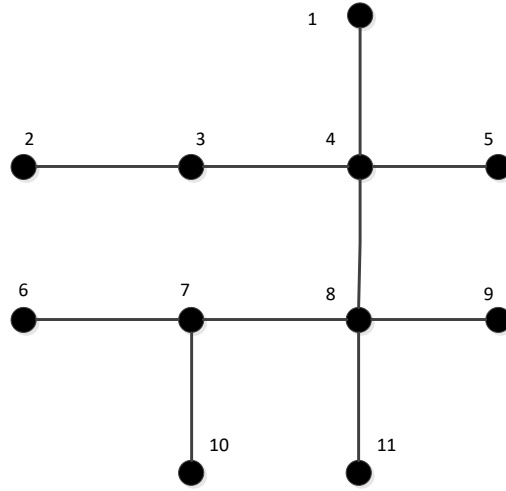


Fig.7 Sample 11 node test feeder

TABLE II. 11-NODE TEST FEEDER LINE SEGMENT DATA

Node		Length (m)
from	to	
4	3	228
4	5	228
3	2	136
1	4	220
7	10	180
4	8	913
8	7	136
8	11	182
7	6	136
8	9	228

TABLE III. PMU PLACEMENT SOLUTIONS FOR 11-NODE TEST FEEDER

solution	PMU installation nodes
1	2,4,7,8
2	3,4,7,8

The proposed simultaneous optimization problem is solved by BCGA for co-optimal placement of PMUs and wireless sensors considering two different reliability values:  $T=0.6$  and  $T=1$ , in which the former denotes a weak reliability level while the latter enforces the maximum level of reliability.

Since there are 171 areas in the studied mesh grid, each chromosome within the population (CR vector) has 171 genes. The results of the optimization problem for this network are shown in Table 4. In this case studies it is assumed that the price of each PMU is about \$40000 [18] and the price of each PPSN and EHSN is about \$120 and \$150, respectively.

The cost of communication infrastructure for each router node is assumed to be \$150. Therefore the price of each PPSN and EHSN and related infrastructure is regulated as \$120 and \$300, respectively.

TABLE IV. OPTIMIZATION PROBLEM RESULTS

<i>Case 1. Reliability coefficient (<math>T=0.6</math>)</i>				
<i>solution</i>	<i>node # (PMU is installed)</i>	<i>node # (PPSN is installed)</i>	<i>Area # (EHSN is installed)</i>	<i>Total cost (in US \$)</i>
1	2,4,7,8	2,4,7,8	38,40,60,78,96,104,112,130	162880
2	3,4,7,8	3,4,7,8	40,58,76,94,112,130	162280
<i>Case 2. Reliability coefficient (<math>T=1</math>)</i>				
<i>solution</i>	<i>node # (PMU is installed)</i>	<i>node # (PPSN is installed)</i>	<i>Area # (EHSN is installed)</i>	<i>Total cost (in US \$)</i>
1	2,4,7,8	2,4,7,8	47,50,57,58,66,75,85,93,102,112,121,131,140	164380
2	3,4,7,8	3,4,7,8	49,50,58,68,76,84,94,103,113,124,131,140	164080

As can be seen, the second solution in each case is the best in terms of the minimum total cost. Higher reliability level (case 2), necessitates an increased number of EHSNs which in turn increases the total cost of investment. A schematic of solution 4 is illustrated in Figure 8.

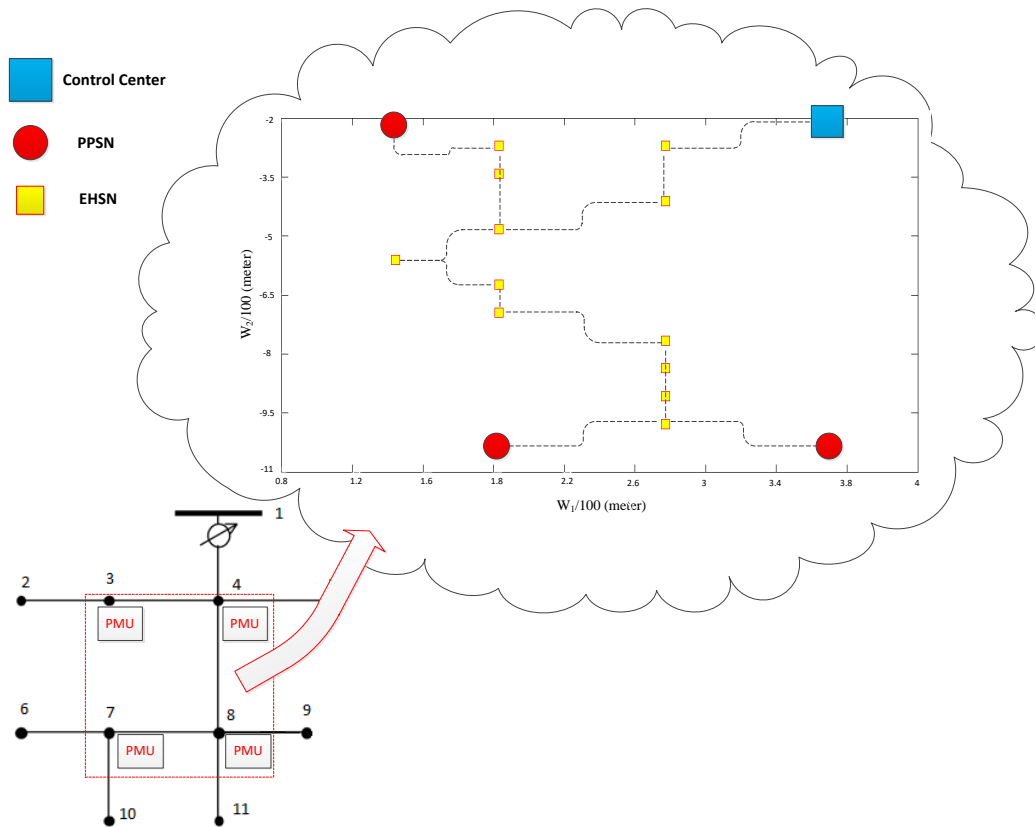


Fig. 8 Optimal PMU and related communication devices placement in a 11-node test system with high-reliability level

## 4.2. IEEE 37 NODE TEST FEEDER

The IEEE 37 node test feeder is shown in Figure 9. For more simplicity, the nodes are numbered from 1 to 37. Similar to the previous study, two cases are realized based on different reliability levels:  $T=0.6$  and  $T=1$ . In this case, transmitting range ( $tr$ ) of each PPSN and EHSN is assumed to be 150 meters. According to the IEEE segment data, the mesh grid size can be calculated as  $d=70.17$ ,  $n_1=11$ ,  $n_2=24$ , and  $N_T=264$ . The simulation results are listed in Table 5. The control center is chosen at node 37.

It can be seen that in the first case, 16 EHSNs are needed for full observability of the communication system while 31 sensors are needed to do so in the second case. Again, as observed from simulation results, higher reliability levels put more financial burdens on the system planner.

TABLE V. OPTIMIZATION PROBLEM RESULTS

<i>Case 1. Reliability coefficient (<math>T=0.6</math>)</i>			
<i>node # (PMU is installed)</i>	<i>node # (PPSN is installed)</i>	<i>Area # (EHSN is installed)</i>	<i>Total cost (in US \$)</i>
2,4,8,11,14,18,22,24,31, 32, 35,37	2,4,8,11,14,18,22,24,31,32 35,37	26,48,66,77,87,92,96,104,108,114, 136,158,192,214,224,236	486240
<i>Case 2. Reliability coefficient (<math>T=1</math>)</i>			
<i>node # (PMU is installed)</i>	<i>node # (PPSN is installed)</i>	<i>Area # (EHSN is installed)</i>	<i>Total cost (in US \$)</i>
2,4,8,11,14,18,22,24,31, 32,35,37	2,4,8,11,14,18,22,24,31,32 35,37	15,27,34,37,39,43,47,51,53,58,62,63,75,81,87,93,99,103,113 ,123,146,156,168,190,202,211,212,213,225,237,248	490740

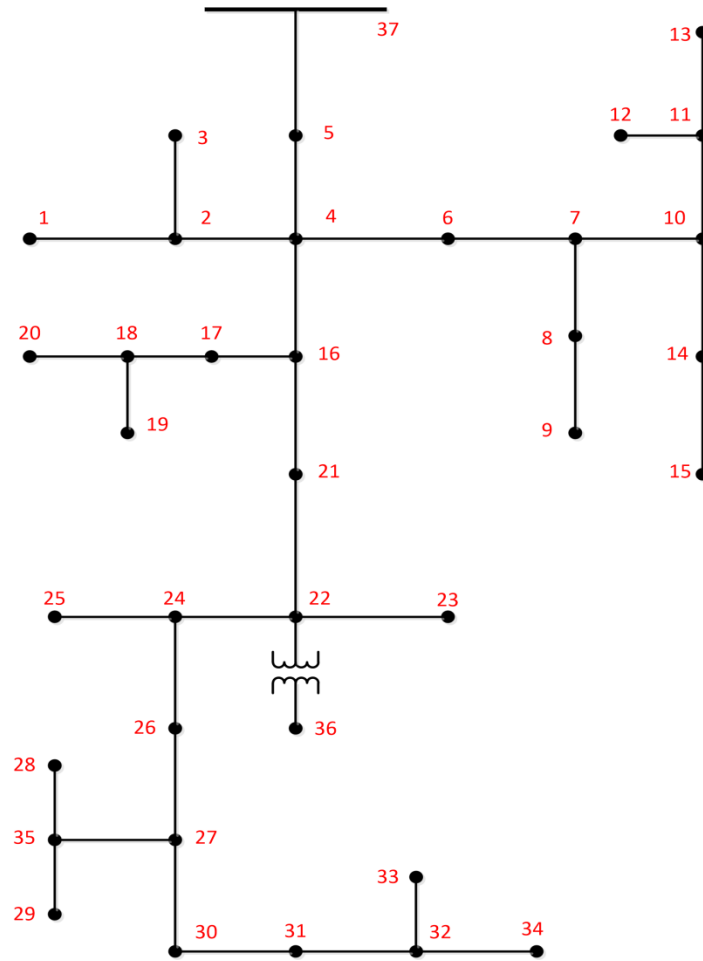


Fig. 9 IEEE 37 node test feeder

To get more insight into the performance of the proposed algorithm, convergence of the solution method in IEEE 37 node test feeder is shown in Figure 10. As can be seen, after about 90 iterations, the BCGA can reach a solution. The computational time for 90 iteration is 1492.6s. For the 11 node test system, this takes a lower number of iterations (about 50). It should be noted that due to the simultaneous optimization of the power and communication systems, the calculation burden is increased as the system size gets bigger. However, as there is no need to make a plan in real-time (or near real-time), off-line programming could be used.

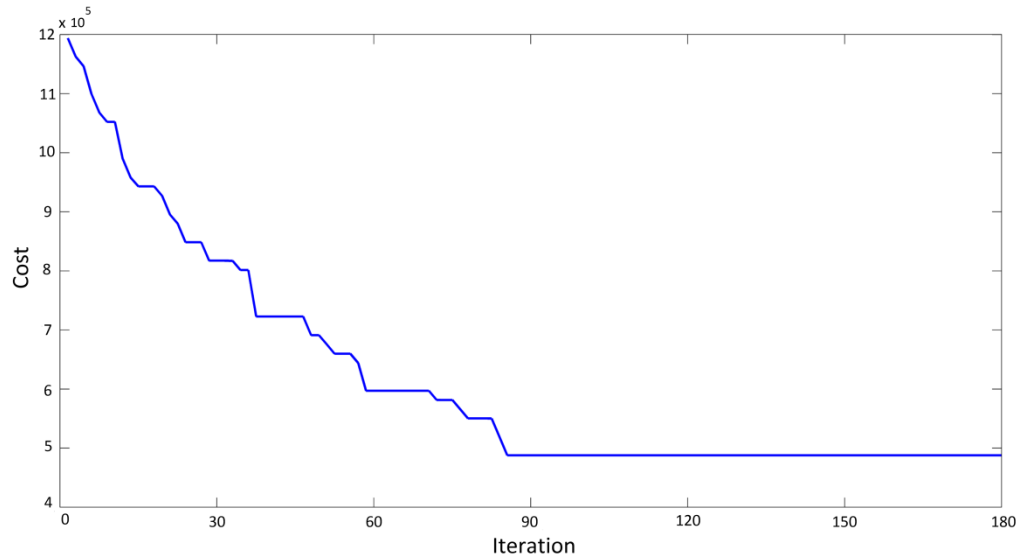


Fig. 10 Convergence of proposed method on IEEE 37 node test feeder

The results of case studies showed that implementing the hybrid wireless sensors as communication infrastructure with PMU placement simultaneously can decrease the cost of the communication system and PMU installation. The CI cost in this infrastructure is much lower than the PMU installation. For example, in the 37 node test system, the PMU placement cost in case 2 is \$480,000. However, the communication infrastructure cost is \$10740. The results of [18] show that the optic fiber cables cost is considerable compared to PMU cost. Respect to the low cost of CI in this system, it is better than the high reliable case is chosen. As can be seen in Table 5, the difference between case 1 and 2 is just \$4500. Compare to the total price of the network, and according to the high importance of reliability, it is more reasonable that the second case (higher reliability coefficient) in each sample network is chosen.

The low cost and convenient infrastructure of wireless sensors increase the incentive for more researches about the capability and feasibility of wireless sensors as a communication link for transferring data in the power system. So, more researches about the bandwidth and data transferring capability of the high-tech wireless sensors in this field are vital.

## 5. Conclusion

This paper presented a novel co-optimal PMU-sensor placement strategy in power systems considering observability and reliability constraints. Wireless sensors were used as communication links in the wide area measurement system. The applicability and effectiveness of this communication system were evaluated through different case studies. The main advantage of the proposed system was twofold: low cost and simple infrastructure. It was demonstrated through computer simulations that using high-level wireless sensors could help proper PMUs data transferring. The results also showed that this novel communication structure for the power system could be useful for diminishing the required installation cost. The results showed that by using the proposed method, the communication infrastructure cost is much lower than the PMU installation cost which is competitive to the conventional fiber-link optic solutions with considerably higher costs.

For better performance of the system, reliability and security of wireless sensors are significant. So in future works, these issues will be considered. Also, the bandwidth capacity of the wireless sensor for proper data transferring in the power network will be further analyzed.



## REFERENCES

- [1] R. Dubey, M. Popov and J. d. J. C. Muro, "Cost Effective Wide Area Measurement Systems for Smart Power Network," *IEEE Power and Energy Technology Systems Journal*, vol. 5, no. 3, pp. 85 - 93, 2018.
- [2] H.-Y. Su, F.-M. Kang and C.-W. Liu, "Transmission Grid Secondary Voltage Control Method Using PMU Data," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2908 - 2917, 2018.
- [3] A. Bashian, M. Hojat, M. H. J. DashtBayaz and H. Golmohamadi, "Security-Based Tariff for Wheeling Contracts Considering Fair Congestion Cost Allocation," *Journal of Control, Automation and Electrical Systems*, vol. 25, no. 3, pp. 368-380, 2014.
- [4] D. Falabretti, M. Delfanti and M. Merlo, "Distribution networks' observability: A novel approach and its experimental test," *Sustainable Energy, Grids and Networks*, vol. 13, pp. 56-65, March 2018.
- [5] L. N. Yiming Wu, "Adaptive data link configuration for WAMC applications using a Stateful Data Delivery Service platform," *Sustainable Energy, Grids and Networks*, vol. 6, pp. 166-175, June 2016.
- [6] A. Ghasemkhani, H. Monsef, A. Rahimi-Kian and A. Anvari-Moghaddam, "Optimal Design of a Wide Area Measurement System for Improvement of Power Network Monitoring Using a Dynamic Multiobjective Shortest Path Algorithm," *IEEE Systems Journal*, vol. 11, no. 4, pp. 2303 - 2314, 2017.
- [7] P. Castello, C. Muscas, P. A. Pegoraro and S. Sulis, "Active Phasor Data Concentrator performing adaptive management of latency," *Sustainable Energy, Grids and Networks*, vol. 16, pp. 270-277, December 2018.
- [8] A. Bashian, T. Sharifian, M. H. Javidi and M. Hojat, "Determination of tariff for wheeling contracts considering fairness congestion cost allocation," in *European Conference on Power and Energy Systems IASTED*, Crete, Greece, , August 2011.
- [9] S. Zhu, L. Wu, S. Mousavian and J. H. Roh, "An optimal joint placement of PMUs and flow measurements for ensuring power system observability under N-2 transmission contingencies," *International Journal of Electrical Power & Energy Systems*, vol. 95, pp. 254-265, 2018.
- [10] A. Asgari and K. G. Firouzjah, "Optimal PMU placement for power system observability considering network expansion and N - 1 contingencies," *IET Generation, Transmission & Distribution*, vol. 12, no. 18, pp. 4216 - 4224, 2018.
- [11] S. Nikkhah, J. Aghaei, B. Safarinejadian and M.-A. Norouzi, "Contingency constrained phasor measurement units placement with n - k redundancy criterion: a robust optimisation approach," *IET Science, Measurement & Technology*, vol. 12, no. 2, pp. 151 - 160, 2018.
- [12] M. Dalali and H. K. Karegar, "Optimal PMU placement for full observability of the power network with maximum redundancy using modified binary cuckoo optimisation algorithm," *IET Generation, Transmission & Distribution*, vol. 10, no. 11, pp. 2817 - 2824, 2016.
- [13] N. Xia, H. Gooi, S. Chen and M. Wang, "Redundancy based PMU placement in state estimation," *Sustainable Energy, Grids and Networks*, vol. 2, pp. 23-31, June 2015.
- [14] J. Huang, N. E. Wu and M. C. Ruschmann, "Data-Availability-Constrained Placement of PMUs and Communication Links in a Power System," *IEEE Systems Journal*, vol. 8, no. 2, pp. 483 - 492, 10 October 2013.
- [15] A. Ghasemkhani, A. Anvari-Moghaddam, J. M. Guerrero and B. Bak-Jensen, "An efficient multi-objective approach for designing of communication interfaces in smart grids," in *2016 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe)*, Ljubljana, Slovenia , Oct. 2016 .
- [16] D. W. Eniye Tebekaemi, "Secure overlay communication and control model for decentralized autonomous control of smart micro-grids," *Sustainable Energy, Grids and Networks*, vol. 18, June 2019.
- [17] M. Shahraeini, M. S. Ghazizadeh and M. H. Javidi, "Co-Optimal Placement of Measurement Devices and Their Related Communication Infrastructure in Wide Area Measurement Systems," *IEEE TRANSACTIONS ON SMART GRID*, vol. 3, no. 2, p. 684-691, 2012.
- [18] M. B. Mohammadi, R.-A. Hooshmand and F. H. Fesharaki, "A New Approach for Optimal Placement of PMUs and Their Required Communication Infrastructure in Order to Minimize the Cost of the WAMS," *IEEE Transactions on Smart Grid* , vol. 7, no. 1, pp. 84 - 93, Jan. 2016.
- [19] M. A. Cruz, H. R. Rocha and M. H. Paiva, "An algorithm for cost optimization of PMU and communication infrastructure in WAMS," *International Journal of Electrical Power & Energy Systems*, vol. 106, pp. 96-104, March 2019.
- [20] X. Zhu, M. H. F. Wen, V. O. K. Li and K.-C. Leung, "Optimal PMU-Communication Link Placement for Smart Grid Wide-Area Measurement Systems," *IEEE Transactions on Smart Grid*, pp. 1 - 1, 27 July 2018.
- [21] B. Appasani and D. K. Mohanta, "Co-Optimal Placement of PMUs and Their Communication Infrastructure for Minimization of Propagation Delay in the WAMS," *IEEE Transactions on Industrial Informatics*, vol. 14, no. 5, pp. 2120 - 2132, 30 January 2018.
- [22] N. P. Theodorakatos, "Optimal Phasor Measurement Unit Placement for Numerical Observability Using Branch-and-Bound and a Binary-Coded Genetic Algorithm," *Electric Power Components and Systems*, vol. DOI:10.1080/15325008.2019.1605635, p. 1-15, May 2019.
- [23] N. Theodorakatos, "Optimal Phasor Measurement Unit Placement for Numerical Observability Using A Two-Phase Branch-and-Bound Algorithm," *International Journal of Emerging Electric Power Systems*, vol. 19, no. 3, pp. 1-25, 2018.
- [24] N. Theodorakatos, N. Manousakis and G. Korres, "Optimal placement of PMUS in power systems using binary integer programming and genetic algorithm," in *MedPower 2014*, Athens, Greece , Nov. 2014 .
- [25] G. N. Korres, N. M. Manousakis, T. C. Xygkis and J. Löfberg, "Optimal phasor measurement unit placement for numerical observability in the presence of conventional measurements using semi-definite programming," *IET Generation, Transmission & Distribution* , vol. 9, no. 15, pp. 2427 - 2436, November 2015.

- [26] M. Moradi-Sepahvand, E. Mashhour and S. S. o. Mortazavi, "Optimal placement of a combination of single-phase and three-phase  $\mu$ PMUs for observability of smart distribution networks with asymmetrical structure," *International Journal of Electrical Power & Energy Systems*, vol. 105, pp. 592-601, February 2019.
- [27] T. C. Xygkis, G. N. Korres and N. M. Manousakis, "Fisher Information-Based Meter Placement in Distribution Grids via the D-Optimal Experimental Design," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 1452 - 1461, July 2016.
- [28] A. B. Noel, A. Abdaoui, T. Elfouly, M. H. Ahmed and A. Badawy, "Structural Health Monitoring Using Wireless Sensor Networks: A Comprehensive Survey," *IEEE Communications Surveys & Tutorials*, vol. 19, no. 3, pp. 1403 - 1423, 2017.
- [29] H. Jung and I.-H. Lee, "Secrecy Performance Analysis of Analog Cooperative Beamforming in Three-Dimensional Gaussian Distributed Wireless Sensor Networks," *IEEE Transactions on Wireless Communications*, vol. 18, no. 3, pp. 1860 - 1873, 2019.
- [30] J. Horneber and A. Hergenröder, "A Survey on Testbeds and Experimentation Environments for Wireless Sensor Networks," *IEEE Communications Surveys & Tutorials*, vol. 16, no. 4, pp. 1820 - 1838, 2014.
- [31] P. Park, S. C. Ergen, C. Fischione, C. Lu and K. H. Johansson, "Wireless Network Design for Control Systems: A Survey," *IEEE Communications Surveys & Tutorials*, vol. 20, no. 2, pp. 978 - 1013, 2018.
- [32] M. Ayaz, M. Ammad-uddin, I. Baig and e.-H. M. Aggoune, "Wireless Sensor's Civil Applications, Prototypes, and Future Integration Possibilities: A Review," *IEEE Sensors Journal*, vol. 18, no. 1, pp. 4 - 30, 2018.
- [33] S.-s. Chiang, C.-h. Huang and K.-c. Chang, "A Minimum Hop Routing Protocol for Home Security Systems Using Wireless Sensor Networks," *IEEE Transactions on Consumer Electronics*, vol. 53, no. 4, pp. 1483 - 1489, 2007.
- [34] V. Agarwal, R. A. DeCarlo and L. H. Tsoukalas, "Modeling Energy Consumption and Lifetime of a Wireless Sensor Node Operating on a Contention-Based MAC Protocol," *IEEE Sensors Journal*, vol. 17, no. 16, pp. 5153 - 5168, 2017.
- [35] M. El-Sherif, Y. Fahmy and H. Kamal, "Lifetime maximisation of disjoint wireless sensor networks using multiobjective genetic algorithm," *IET Wireless Sensor Systems*, vol. 8, no. 5, pp. 200 - 207, 2018.
- [36] A. E. Zonouz, L. Xing, V. M. Vokkarane and Y. Sun, "Hybrid wireless sensor networks: A reliability, cost and energy-aware approach," *IET Wireless Sensor Systems*, vol. 6, no. 2, pp. 42 - 48, 21 April 2016.
- [37] A. E. Zonouz, L. Xing, V. M. Vokkarane and Y. Sun, "Application communication reliability of wireless sensor networks," *IET Wireless Sensor Systems*, vol. 5, no. 2, pp. 58 - 67, 23 April 2015.
- [38] M. A. Mahmood, W. K.G.Seah and I. Welch, "Reliability in wireless sensor networks: A survey and challenges ahead," *Computer Networks*, vol. 79, pp. 166-187, 14 March 2015.
- [39] C. Wang, L. Xing and V. M. Vokkarane, "Reliability and lifetime modeling of wireless sensor nodes," *Microelectronics Reliability*, vol. 54, no. 1, pp. 160-166, January 2014.
- [40] J.-H. Park, "Time-dependent reliability of wireless networks with dependent failures," *Reliability Engineering & System Safety*, vol. 165, pp. 47-61, September 2017.
- [41] A. Bidadfar, H. Hooshyar and L. Vanfretti, "Dynamic Thévenin equivalent and reduced network models for PMU-based power system voltage stability analysis," *Sustainable Energy, Grids and Networks*, vol. 16, pp. 126-135, December 2018.
- [42] L.-A. Lee and V. Centeno, "Comparison of  $\mu$ PMU and PMU," in *Clemson University Power Systems Conference (PSC)*, Charleston, SC, USA, USA, Sept. 2018 .
- [43] R.Shirkhani, M.Assili and M.Hajiabadi, "observability analysis of distribution network using wireless sensor," master of science thesis, shahrood university of technology, 2016.
- [44] Hougen, P. A. Diaz-Gomez and D. F, "Initial population for genetic algorithms: A metric approach," in *international conference on genetic and evolutionary methods*, January 2007.
- [45] S. Kumar, B. Tyagi, V. Kumar and S. Chohan, "Incremental PMU placement considering reliability of power system network using analytical hierarchical process," *IET Generation, Transmission & Distribution*, vol. 12, no. 16, pp. 3900 - 3909, August 2018.
- [46] N. Xia and H. Gooi, "Observability analysis of PMU placement: Multiple-solution studies," in *IEEE PES Innovative Smart Grid Technologies*, Perth, WA, Australia , 2011.
- [47] E. Makram, Z. Zhao and A. Girgis, "An improved model in optimal PMU placement considering sensitivity analysis," in *IEEE/PES Power Systems Conference and Exposition*, Phoenix, AZ, USA , March 2011.